

Properties of Solutions of a Riccati Matrix Differential Equation

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1. Introduction. Recently the author [16] extended to Riccati matrix differential equations certain results of REDHEFFER [9; 10] on scalar Riccati equations. In a subsequent paper, REDHEFFER [11] pointed out that the conclusions of [16] did not provide a complete generalization of his earlier results for the scalar case, and proceeded to obtain a full generalization with the ingenious aid of algebraic results on “*-products” of matrices. Moreover, REDHEFFER answered a question raised by the author [16; p. 229] on a necessary and sufficient condition for solutions to possess a certain dissipative property.

In the present paper it is shown that the analytic methods of REID [16] yield properties of solutions of the type presented by REDHEFFER [11]. Section 2 is devoted to a comparison of the earlier results of the author [16] and REDHEFFER [11] and with statements of Theorems 2.1 and 2.2, which present the principal results of the paper. The result of Theorem 2.1 is essentially that of Theorem III of REDHEFFER [11]. Conclusion (i) of Theorem 2.2 corresponds to the first conclusion of Theorem XII of REDHEFFER [11], and in statement it is a direct extension and improvement of the result of Theorem 4.1 of REID [16]. The proof of this result involves an extension of a quadratic form problem which was considered a number of years ago by various authors (see ALBERT [1], DINES [3], FINSLER [4], HESTENES & McSHANE [6], and REID [13]); because of the intrinsic interest of the hermitian form problem, there are presented in Section 6 certain general results which include as a special instance the needed result of Theorem 5.1. Conclusion (ii) of Theorem 2.2 presents a corrected form of the final conclusion of Theorem XII of REDHEFFER [11], which is in error. It is to be noted that certain of the results appearing in the lemmas of Section 3 have been cast in a form to establish the existence of solutions, rather than to postulate their existence as in REID [16] and REDHEFFER [11]; in this respect

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