

On Perturbations of Periodic Solutions

YASUTAKA SIBUYA¹

Communicated by A. ERDÉLYI

INTRODUCTION

Studying the problem of perturbation of a system of linear ordinary differential equations, E. A. CODDINGTON & N. LEVINSON gave criteria for the existence of periodic solutions in the case when the system of first variation has nontrivial periodic solutions [3]. Recently the author of the present paper obtained a method of constructing periodic solutions [9]. By using this method we shall give similar criteria in the case of perturbation of a nonlinear system under the situation similar to theirs.² We shall discuss this problem in Chapter I. In Chapter II, we shall give criteria for the stability of the periodic solutions which will be constructed in Chapter I. Our criteria for the stability are quite similar to those given by I. I. BLEKMAN [1, 2] and J. A. NOHEL [7].

I. EXISTENCE OF PERIODIC SOLUTIONS

1. Assumptions. We shall consider a system of nonlinear ordinary differential equations:

$$(1.1) \quad \frac{dy_i}{dt} = f_i(t, y, \mu) \quad (j = 1, 2, \dots, n),$$

where t is a real variable, μ a real parameter and the f_j are functions continuous in t, y, μ , sufficiently smooth with respect to y, μ , and periodic in t of period 2π .

Let us suppose that the unperturbed system

$$(1.2) \quad \frac{dy_i}{dt} = f_i(t, y, 0) \quad (j = 1, 2, \dots, n)$$

has a periodic solution of period 2π :

$$(1.3) \quad y_i = \phi_i(t) \quad (j = 1, 2, \dots, n).$$

¹ The author is a Fulbright grantee. This work is mainly based on discussions with Professor E. A. CODDINGTON at the University of California, Los Angeles, and the author's technical report at the Mathematics Research Center, U.S. Army, University of Wisconsin [10]. This paper was prepared at the Massachusetts Institute of Technology under the sponsorship of the Office of Naval Research.

² The study in this direction is not new. For example, see D. C. LEWIS [5], W. S. LOUD [6].