

*On an Extension of Cesàro's Methods of Summability to Logarithmic Scales**

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1. The arithmetic method $T(A_n)$. The methods we are going to consider are, like Cesàro's methods, special cases of Nörlund's (sometimes called Nörlund-Voronoi) methods of summability, and we begin by recalling basic definitions and properties of the latter. For the literature of the subject see section 6 below.

Consider a sequence of numbers a_0, a_1, a_2, \dots ; let $A_n = a_0 + a_1 + \dots + a_n$. A sequence s_n ($n = 0, 1, 2, \dots$) is said to be limitable by the method $T(A_n)$ to the limit s if

$$\lim_{n \rightarrow +\infty} \frac{a_n s_0 + a_{n-1} s_1 + \dots + a_0 s_n}{A_n} = s,$$

and an infinite series $\sum_0^\infty u_n$ is said to be summable by the method $T(A_n)$ to the sum s , if the sequence of its partial sums $s_n = \sum_0^n u_n$ is limitable $T(A_n)$ to s . We assume that the A_n satisfy the following conditions:

I. The numbers A_n , with the possible exception of a finite number, are strictly positive, *i.e.*, $A_n > 0$ for $n > n_0$.

II.
$$\lim_{n \rightarrow \infty} \frac{a_n}{A_n} = 0, \quad \text{i.e.,} \quad \lim_{n \rightarrow \infty} \frac{A_{n-1}}{A_n} = 1.$$

III.
$$\sum_{n=0}^{\infty} A_n = +\infty.$$

These conditions are necessary if every convergent sequence is to be limitable $T(A_n)$ to the same limit. In some theorems we impose on the A_n some additional conditions, for example

IV.
$$\lim_{n \rightarrow \infty} A_n = +\infty.$$

Let

$$\sum_{n=0}^{\infty} A_n x^n = \phi(x).$$

* Summability $(C, \alpha_0, \alpha_1, \dots, \alpha_k)$ was introduced in ZYGMUND [12], who also stated without proof a number of results. I am grateful to Professor ZYGMUND for the suggestion to develop the topic and for his helpful advice. The present paper is an abbreviated version of a PhD thesis submitted in September 1955 to the Department of Mathematics of the University of Chicago.