

A Variational Principle for Convection of Heat

Part I

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Part II

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1. Introduction. The equations of heat transfer can be expressed in the variational language in a variety of ways. A variational principle for the equation of heat conduction in which only the time derivative of the temperature is varied, the temperature itself remaining unchanged, has been given by CHAMBERS [1]. ROSEN [2] has formulated a variational principle for diffusional processes which is a modification of ONSAGER's principle of minimum dissipation. In his formulation the time derivative of the temperature is held fixed while the temperature is varied. Recently BIOT [3] has developed a variational principle for the equation of heat conduction using the concepts of thermal potential, dissipation function and generalized thermal force. It appears to be most general and capable of dealing with non-linear and surface effects. It fits into a unified thermodynamic theory, embracing a large category of physical phenomena, leading to the equations of the Lagrangian type for the thermal flow field.

In Part I of this paper the variational principle of BIOT is extended to *transient* heat transfer in fluids under forced flow including the heat generated by viscous dissipation. In Part II a similar variational principle is developed for the problems of *steady* heat transfer in channel flows for large PÉCLER numbers (axial heat conduction neglected). The method is applied to heat transfer between two semi-infinite parallel plates for (i) slug flow and (ii) parabolic flow. It is found that the temperature distribution is in good agreement with that obtained by "orthogonal functions expansion method" (SCHNEIDER [4] and V. D. DOES & SCHENK [5]).

2. The variational principle. Consider an isotropic medium with thermal conductivity $k(x,y,z)$ and heat capacity $c(x,y,z)$ per unit volume. H being the