

Remarks on Stefan-Type Free Boundary Problems for Parabolic Equations

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Introduction. Free boundary problems for parabolic equations started to receive rigorous treatment in the last decade. Various methods have been devised to obtain existence and uniqueness theorems. We refer in particular to works (on Stefan problems) of J. DOUGLAS [1], I. I. KOLODNER [5], W. T. KYNER [6], W. I. MIRANKER [7], L. I. RUBINSTEIN [9] and G. SESTINI [10], [11].

Recently we have investigated various free boundary problems from a unified point of view. The first step in solving each problem consists in reducing it (by a unified technique) to a nonlinear integral equation of Volterra type $v = Tv$ (T nonlinear). The existence of a unique solution for *all* future times has been proved for a variety of Stefan problems [2], for a drop in gas [3] and for a bubble in liquid [4]. The method of proving existence and uniqueness in these papers consists of two steps: (i) a proof that T is locally a contraction, (ii) construction step-by-step of a solution in the large, deriving first appropriate *a priori* inequalities. There are however Stefan problems for which it is not known to us whether T is locally a contraction or even bounded. Such a problem arises physically if there is initially no water (only ice). In the first part of this paper we prove an existence theorem for such a problem.

In [3], [4] we have also derived the asymptotic behavior of the solution as the time t tends to infinity. In the second part of this paper we derive asymptotic formulas ($t \rightarrow \infty$) for two Stefan problems. The method has a general feature in common with the methods of [3], [4], namely, it reduces the problem of evaluating the free boundary $s(t)$ to that of estimating a certain integral. Physically, this integral measures how much heat is wasted in heating the water instead of melting the ice. The crucial step is to obtain a sharp estimate of this integral. The method used in this paper for estimating it is more effective (for the present problems) than the method of [3], [4].

We give a brief summary of the paper. In §1 we prove an existence theorem for the Stefan problem (1.1)–(1.4). The proof is given in 1.2 and various generali-

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