

Stationary Markov Chains and Independent Random Variables¹

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Introduction. The object of this paper is to obtain a necessary and sufficient condition for a stationary Markov chain $\{x_n\}$, $n = 0, \pm 1, \dots$, with an enumerable number of states to have a representation of the form

$$(1) \quad x_n = g(\alpha_n, \alpha_{n-1}, \dots),$$

where g is a Borel measurable function and $\{\alpha_n\}$, $n = 0, \pm 1, \dots$, is a process of independent uniformly distributed random variables $[0, 1]$. The condition is that $\{x_n\}$ be ergodic and have no periodic states. The proof makes use of ideas and techniques in [2].

Preliminary Remarks. Let $\{x_n\}$, $n = 0, \pm 1, \dots$, be a stationary Markov chain with an enumerable number of states which can be conveniently labeled by the integers $1, 2, \dots$. The probability structure of the chain is characterized by its one step transition probability matrix

$$P = (p_{ij}; i, j = 1, 2, \dots)$$

$$p_{ij} = P(x_{n+1} = j \mid x_n = i)$$

and its instantaneous stationary row vector of probabilities

$$p = (p_i; i = 1, 2, \dots)$$

$$p_i = P(x_n = i).$$

The instantaneous vector of probabilities is a left invariant vector of P (see [1]); that is to say,

$$pP = p.$$

Lemma 1 of [2] already indicates that a representation of the form (1) is possible only if the process $\{x_n\}$ is ergodic and has no periodic states. We have therefore to show that such a representation does exist if the process $\{x_n\}$ is ergodic and has no periodic states.

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